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Conformal invariance of the pure spinor superstring in a curved background

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Abstract: It is shown that the pure spinor formulation of the heterotic superstring in a generic gravitational and super Yang-Mills background has vanishing one-loop beta functions.

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1. Introduction

It is a well known fact that the quantum mechanical preservation of the conformal symmetry in string theory implies that the background space-time satisfies suitable equations of motion. In the simplest case, a bosonic string coupled to a curved space-time geometry is described by a conformal field theory if the space-time metric satisfies the Einstein equations plus α'-corrections.

The instabilities produced by the presence of the tachyon can be avoided if we include supersymmetry. There are two traditional ways to achieve this end in string theory. The first possibility considers susy at world-sheet level, so one constructs the so called Ramond-Neveu-Schwarz (RNS) superstring. The second possibility adds susy at the space-time level, so one describes the dynamics of the superstring by using the Green-Schwarz (GS) model.

There are various problems in these approaches that lead to think in an alternative description of the superstring. We mention some of them. On the one hand, it is difficult to describe space-time fermions in the RNS model and, therefore, backgrounds with non trivial RR fields. On the other hand, the lack of a covariant quantization of the GS superstring remains an open problem.

The alternative approach to describe a superstring is the recently developed Berkovits formalism [1]. Here, the quantization is performed by constructing a BRST charge $Q = \int \lambda^\alpha d_\alpha$, where λ^α is a pure spinor[1] and d_α is the world-sheet generator of superspace

[1]It is a c-number field constrained to satisfy the pure spinor condition $(\lambda\gamma^a\lambda) = 0$ with γ^a being the 16×16 symmetric ten-dimensional gamma matrices.

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translations. It has been possible to verify that the cohomology of Q produces the correct superstring spectrum, in the light-cone gauge [2] and in a manifestly ten-dimensional super-Poincare covariant manner [3]. The model can be also formulated to describe the coupling of the superstring in a background with RR fields, such as the $\mathrm{AdS}_5 \times \mathrm{S}^5$ [4], where one-loop conformal invariance has been proved [5].

Let us be more precise in the case relevant for the present paper. Consider a pure spinor description of the heterotic superstring. It is shown in [6] that the nilpotence of the BRST charge and the holomorphicity of the BRST current put the background on-shell, at least in the classical level. The next step is to study this theory at the quantum level. We fix the world-sheet surface to be a sphere, that is we are considering string perturbation theory at the tree-level. But we can make a quantum field theory for the sigma model and so construct a loop expansion on the sphere. In this way we can verify if the BRST charge remains nilpotent and if the BRST charge remains holomorphic. These conditions are related to the conformal invariance of the model at the quantum level. The argument for this is given in [6] for the linearized case. Of course, it would be interesting to study the non-linear version of this. But we decide to study the conformal invariance of the sigma model at the one-loop level by computing the beta functions.

In an off-shell formulation of the $N = 1$ supergravity[2] one might interpret the beta functions of the first two couplings in the sigma model action of (2.1) as the equations of motion for two independent off-shell superfields which solve the supergravity Bianchi indentities. On the SYM side one cannot find such coupling since there is no known off-shell superfield which solves the SYM Bianchi identities. Since the constrains found by Berkovits and Howe already put the supergravity/SYM system on-shell all the superfields in the background field expansion done in this work are also on-shell.

In section 2 we will review the pure spinor sigma model for the heterotic superstring in a generic background field and the constraints imposed by the nilpotence of the BRST charge and the holomorphicity of the BRST current. In section 3 we will perform a covariant background field expansion of the sigma model action and in section 4 will compute the one-loop effective action. In section 5 we will show that the one-loop beta functions vanish due the classical superspace constraints of [6].

2. The pure spinor sigma model

Let us consider an heterotic string in a curved background. The action in the pure spinor formalism is given by

$$\begin{aligned} S = \frac{1}{2\pi\alpha'} \int d^2 z \Big[& \frac{1}{2}\Pi^a \overline{\Pi}^b \eta_{ab} + \frac{1}{2}\Pi^A \overline{\Pi}^B B_{BA} + \Pi^A \overline{J}^I A_{IA} + \\ & + d_\alpha (\overline{\Pi}^\alpha + \overline{J}^I W_I^\alpha) + \lambda^\alpha \omega_\beta (\overline{\Pi}^A \Omega_{A\alpha}{}^\beta + \overline{J}^I U_{I\alpha}{}^\beta) \Big] + S_{\overline{J}} + S_{\lambda,\omega} + S_{FT}\,, \end{aligned} \tag{2.1}$$

where $\Pi^A = \partial Z^M E_M{}^A$, $\overline{\Pi}^A = \overline{\partial} Z^M E_M{}^A$ with $E_M{}^A$ being the supervielbein, $Z^M = (x^m, \theta^\mu)$; $m = 0, \ldots, 9$, $\mu = 1, \ldots, 16$ the superspace coordinates, d_α the world-sheet gen-

[2] See, for example [7].

erator of superspace translations. $S_{\overline{J}}$ is the action for the gauge group variables, $S_{\lambda,\omega}$ is the action for the pure spinor variables (λ, ω). S_{FT} is the Fradkin-Tseytlin term which is given by

$$S_{FT} = \frac{1}{2\pi} \int d^2 z \, r \Phi \,, \tag{2.2}$$

where r is the world-sheet scalar curvature and Φ is a superfield whose θ-independent part is the dilaton. Recall that the Fradkin-Tseytlin term breaks conformal invariance, then it can be seen as an α'-correction which restores it at the quantum level.

Since the pure spinor variables can only enter in the combinations $J = \lambda^\alpha \omega_\alpha$ (the ghost number current) and $N^{ab} = \frac{1}{2}(\lambda\gamma^{ab}\omega)$ (the generator for Lorentz rotations of the pure spiror fields), the couplings between the pure spinor variables and the background fields can be written as

$$\begin{aligned} \lambda^\alpha \omega_\beta \overline{J}^I U_{I\alpha}{}^\beta &= J\overline{J}^I U_I + \frac{1}{2} N^{ab} \overline{J}^I U_{Iab} \,, \\ \lambda^\alpha \omega_\beta \overline{\Pi}^A \Omega_{A\alpha}{}^\beta &= J\overline{\Pi}^A \Omega_A + \frac{1}{2} N^{ab} \overline{\Pi}^A \Omega_{Aab} \,. \end{aligned} \tag{2.3}$$

The quantization of this system is determined by studying the BRST operator $Q = \oint \lambda^\alpha d_\alpha$. In a flat background it is easy to show that $Q^2 = 0$ and $\overline{\partial}(\lambda^\alpha d_\alpha) = 0$. In a curved background, the nilpotence of the BRST charge and holomorphicity of the BRST current at the classical level imply that the background fields satisfy the N=1 SUGRA/SYM equations of motion [6]. In fact, the nilpotence determines the constraints

$$\lambda^\alpha \lambda^\beta T_{\alpha\beta}{}^A = \lambda^\alpha \lambda^\beta H_{\alpha\beta A} = \lambda^\alpha \lambda^\beta \lambda^\gamma R_{\alpha\beta\gamma}{}^\delta = \lambda^\alpha \lambda^\beta F_{I\alpha\beta} = 0 \,, \tag{2.4}$$

and from the holomorphicity of the BRST current we obtain

$$\begin{aligned} T_{\alpha(ab)} &= H_{\alpha ab} = T_{a\alpha}{}^\beta = \lambda^\alpha \lambda^\beta R_{a\alpha\beta}{}^\gamma = 0 \,, \\ T_{\alpha\beta a} &= H_{\alpha\beta a} \,, \qquad F_{Ia\alpha} = W_I^\beta T_{\alpha\beta a} \,, \qquad F_{I\alpha\beta} = \frac{1}{2} H_{\alpha\beta\gamma} W_I^\gamma \,, \\ \nabla_\alpha W_I^\beta &= U_{I\alpha}{}^\beta - W_I^\gamma T_{\gamma\alpha}{}^\beta \,, \qquad \lambda^\alpha \lambda^\beta (\nabla_\alpha U_{I\beta}{}^\gamma + R_{\delta\alpha\beta}{}^\gamma W_I^\delta) = 0 \,, \end{aligned} \tag{2.5}$$

where T, R, H and F are defined as follows. The covariant derivative acting on a super p-form with values in the Lie algebra of the gauge group is given by[3]

$$\nabla \Psi^A = d\Psi^A + \Psi^B \Omega_B{}^A - \Psi^A A + (-1)^p A \Psi^A \,, \tag{2.6}$$

where d is the exterior derivative which maps a super p-form into a super (p+1)-form. Note that by acting one more time the covariant derivative on Ψ^A it is obtained

$$\nabla\nabla\Psi^A = \Psi^B R_B{}^A + F\Psi^A - \Psi^A F \,, \tag{2.7}$$

[3] Wedge product between the superfields is assumed.

where the curvatures R and F are given by

$$R_A{}^B = d\Omega_A{}^B + \Omega_A{}^C\Omega_C{}^B = \frac{1}{2}E^D E^C R_{CDA}{}^B\,, \tag{2.8}$$

$$F = dA - AA = \frac{1}{2}E^B E^A F_{IAB}K^I\,.$$

We can also define the three-form field strength

$$H = dB = \frac{1}{6}E^C E^B E^A H_{ABC}\,,$$

where E^A is the vielbein one form, the potential one form is $A = A_I K^I$ with $[K^I, K^J] = f^{IJ}{}_L K^L$ (f's are structure constants of the Lie group). It is also necessary to define the torsion 2-form

$$T^A = \nabla E^A = \frac{1}{2}E^C E^B T_{BC}{}^A\,. \tag{2.9}$$

The curvatures and the torsion are constrained to satisfy the Bianchi identities, $\nabla T^A = E^B R_B{}^A, dH = \nabla F = \nabla R_A{}^B = 0$. In components they read

$$\begin{aligned}
(\nabla T)_{ABC}{}^D &\equiv \nabla_{[A}T_{BC]}{}^D + T_{[AB}{}^E T_{EC]}{}^D - R_{[ABC]}{}^D = 0\,,\\
(\nabla F)_{ABC} &\equiv \nabla_{[A}F_{BC]} + T_{[AB}{}^D F_{DC]} = 0\,,\\
(\nabla R)_{ABCD}{}^E &\equiv \nabla_{[A}R_{BC]D}{}^E + T_{[AB}{}^F R_{FC]D}{}^E = 0\,,\\
(\nabla H)_{ABCD} &\equiv \nabla_{[A}H_{BCD]} + \frac{3}{2}T_{[AB}{}^E H_{ECD]} = 0\,,
\end{aligned} \tag{2.10}$$

where the antisymmetrization is over A, B, C indices in the first three identities, while in the fourth it is on A, B, C, D indices.

It is also noted in [6] that the action (2.1) has two independent local Lorentz invariances. One acts on the bosonic indices of the vielbein as $\delta\Pi^a = \Pi^b\Lambda_b{}^a$ and the other one acts on the fermionic indices of the vielbein as $\delta\Pi^\alpha = \Pi^\beta\,\Sigma_\beta{}^\alpha$ with $\Omega_{A\alpha}{}^\beta$ transforming as a connection under Σ and the remaining variables of the action as covariant objects (e.g. $\delta\lambda^\alpha = \lambda^\beta\Sigma_\beta{}^\alpha$). The action is also invariant under the shift symmetry

$$\delta\Omega_\alpha = (\gamma_a)_{\alpha\beta}h^{a\beta}\,, \quad \delta\Omega_\alpha{}^{ab} = 2(\gamma^{[a})_{\alpha\beta}h^{b]\beta}\,, \quad \delta d_\alpha = \delta\Omega_{\alpha\beta}{}^\gamma\lambda^\beta\omega_\gamma\,, \quad \delta U_{I\alpha}{}^\beta = W_I^\gamma\delta\Omega_{\gamma\alpha}{}^\beta\,.$$

With the help of these invariances, the constraints (2.4), (2.5) and the Bianchi identities (2.10) it is possible to set $H_{\alpha\beta\gamma} = T_{\alpha\beta}{}^\gamma = 0$ and $T_{\alpha\beta}{}^a = \gamma^a_{\alpha\beta}$. It is argued in [6] that these constraints and the Bianchi identities (2.10) imply the SUGRA/SYM equations. We will show that these choices allow to check the vanishing of the one-loop beta functions.

In order to describe correctly the degrees of freedom of the SUGRA/SYM system, the spinorial derivative of the dilaton superfield must be proportional to the connection Ω_α. In fact, the one-loop preservation of $\overline{\partial}(\lambda^\alpha d_\alpha) = 0$ implies [6]

$$\nabla_\alpha\Phi = 4\Omega_\alpha\,, \tag{2.11}$$

which will be crucial to vanish the one-loop beta functions in the subsequent discussion.

3. The covariant background field method

We expand the action (2.1) in a covariant way. We need to expand the superspace variables, the ghosts and the gauge group variables.

3.1 Superspace expansion

We follow the background field method based on normal coordinates (see [8] and references therein).

We define local coordinates around a certain point in superspace Z^M by specifying the value of its tangent along certain geodesics. Normal coordinates are those in which the geodesics looks like a straight line. If we denote by Y^M the value of the tangent to the geodesics at Z^M, then the geodesics points to $Z^M + Y^M$. As it was shown in [8] any tensor defined at $Z + Y$ can be identified with the tensor at Z by

$$T' = e^{Y^A \nabla_A} T\,,$$

which can be iteratively obtained by applying the operator Δ on T defined as

$$\Delta T = [Y^A \nabla_A, T]\,. \tag{3.1}$$

Here we are using coordinates in the orthonormal frame. Recall $Y^A = Y^M E_M{}^A$ and $\nabla_A = E_A{}^M \nabla_M$, where ∇_M is the covariant derivative containing the Christoffel symbol while ∇_A is the covariant derivative containing the spin connection. As it was argued in [8], ΔT is the parallel transportation of the tensor from $Z + Y$ to Z through the geodesics. Note also that the geodesic equation can be written as $\Delta Y^A = 0$.

In order to expand the action in powers of Y we need to know how the operator Δ acts on the different fields of (2.1). If we choose as a tensor T the covariant derivative of a zero form and if we use (2.6) we can read off the action of Δ on the vielbein and the connections. The result is

$$\begin{aligned}
\Delta E_A{}^M &= -(\nabla_A Y^B + Y^C T_{CA}{}^B) E_B{}^M\,,\\
\Delta \Omega_{A\alpha}{}^\beta &= -(\nabla_A Y^B + Y^C T_{CA}{}^B) \Omega_{B\alpha}{}^\beta + Y^B R_{BA\alpha}{}^\beta\,,\\
\Delta A_{IA} &= -(\nabla_A Y^B + Y^C T_{CA}{}^B) A_{IB} + Y^B F_{IBA}\,.
\end{aligned} \tag{3.2}$$

We need to perform the expansion of $\Pi^A = \partial Z^M E_M{}^A$. Note that ∂Z^M is annihilated by Δ since it does not change under parallel transportation. By inverting the first equation in (3.2), one can see that

$$\Delta E_M{}^A = E_M{}^B (\nabla_B Y^A + Y^C T_{CB}{}^A)\,,$$

therefore

$$\Delta \Pi^A = \nabla Y^A - Y^B \Pi^C T_{CB}{}^A = \nabla Y^A + \Pi^B Y^C T_{CB}{}^A\,, \tag{3.3}$$

where we have defined $\nabla Y^A = \Pi^B \nabla_B Y^A$. By doing the same kind of calculation we obtain

$$\Delta(\nabla Y^A) = -Y^D Y^C \Pi^B R_{BCD}{}^A\,.$$

Analogously, for the barred world-sheet fields we obtain

$$\begin{aligned}\Delta\overline{\Pi}^A &= \overline{\nabla}Y^A - Y^B\overline{\Pi}^C T_{CB}{}^A = \overline{\nabla}Y^A + \overline{\Pi}^B Y^C T_{CB}{}^A\,,\\ \Delta(\overline{\nabla}Y^A) &= -Y^D Y^C \overline{\Pi}^B R_{BCD}{}^A\,,\end{aligned}$$

where $\overline{\nabla}Y^A = \overline{\Pi}^B\nabla_B Y^A$. Note that B_{AB}, W_I^α, U_I and U_I^{ab} are tensors, then they can be expanded following the rule (3.1).

We assume that d_α is a fundamental field, its expansion is

$$d = d_0 + \widehat{d}\,,$$

where d_0 is the background value and $\widehat{d}$ is the quantum fluctuation. Since this expansion is independent of the superspace expansion, it is satisfied that $\Delta\widehat{d} = 0$.

3.2 Ghost and gauge group expansion

As in the case of the d world-sheet field, the pure-spinor ghosts can be treated as fluctuations of some background value

$$\lambda = \lambda_0 + \widehat{\lambda}\,, \qquad \omega = \omega_0 + \widehat{\omega}\,,$$

again, this expansion is independent of the superspace expansion, then it is satisfied $\Delta\widehat{\lambda} = \Delta\widehat{\omega} = 0$. We will not enter in the details of the propagator for these fields. Since only the covariant combinations J and N^{ab} enter in the action, we can make an expansion of the form

$$J = J_0 + J_1 + J_2\,, \qquad N^{ab} = N_0^{ab} + N_1^{ab} + N_2^{ab}\,,$$

where each subscript represents the order in the quantum fluctuations. We are interested in OPE's which depend on (J_0, N_0^{ab}). This is because the OPE terms independent of the fields have short distance behavior like $\frac{1}{(z-w)^2}$, which does not contribute to divergences at one loop. And the terms with quantum fluctuations do not enter in the effective action. The only non vanishing OPE of the type above is

$$N_1^{ab}(z)N_1^{cd}(w) \to \frac{1}{(z-w)}\left[-\eta^{a[c}N_0^{d]b}(w) + \eta^{b[c}N_0^{d]a}(w)\right]. \tag{3.4}$$

In the same way, the gauge current can be expanded as

$$\overline{J}^I = \overline{J}_0^I + \overline{J}_1^I + \overline{J}_2^I\,.$$

As before, we need only the OPE[4]

$$\overline{J}_1^I(\bar{z})\overline{J}_1^J(\bar{w}) \to \frac{1}{(\bar{z}-\bar{w})}f^{IJ}{}_K\overline{J}_0^K(\bar{w})\,. \tag{3.5}$$

In the following we will drop the the 0 subindex, then we will denote $d_{0\alpha}$ as d_α, J_0 as J, N_0^{ab} as N^{ab} and $\overline{J}_0^I$ as $\overline{J}^I$.

[4] This OPE can be obtained by realizing the gauge group contribution to the sigma-model by antichiral Majorana spinors $\rho^{\mathcal{A}}$ ($\mathcal{A} = 1, \ldots, 32$) and noting $\overline{J}^I = \frac{1}{2}K^I_{\mathcal{AB}}\rho^{\mathcal{A}}\rho^{\mathcal{B}}$, where K^I are the gauge group generators. Then we expand these fermions as $\rho^{\mathcal{A}} = \rho_0^{\mathcal{A}} + \widehat{\rho}^{\mathcal{A}}$.

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4. The one-loop effective action

Now we perform the expansion of the action (2.1) in the way described in the previous section with the variables $\mathcal{Q} = (Y, \widehat{\rho}, \widehat{\lambda}, \widehat{\omega})$ as the quantum fluctuations. The effective action will be obtained by integrating out $\mathcal{Q}$. For the one-loop contribution to this action, it is necessary go up to second order in the quantum fluctuations. It is not so difficult to see that up to second order the expansion of the action has the form (we will set $\alpha' = 1$)

$$S = S_0 + \widehat{S} + \widehat{I}\,,$$

where S_0 is the background value of the action,

$$\widehat{S} = \frac{1}{2\pi}\int d^2z\, \frac{1}{2}\eta_{ab}\nabla Y^a \overline{\nabla} Y^b + \widehat{d}_\alpha \overline{\nabla} Y^\alpha + L_g\,, \tag{4.1}$$

determines the propagators for the quantum fluctuations as (3.4), (3.5) and

$$\begin{aligned} Y^a(z,\bar{z})Y^b(w,\bar{w}) &\to -\eta^{ab}\log|z-w|^2\,,\\ \widehat{d}_\alpha(z)Y^\beta(w) &\to \frac{\delta_\alpha^\beta}{(z-w)}\,, \end{aligned} \tag{4.2}$$

note that these terms come from the expansion of the $\Pi^a\overline{\Pi}_a$, $d_\alpha\overline{\Pi}^\alpha$ and L_g terms of (2.1). And

$$\begin{aligned} \widehat{I} = \frac{1}{2\pi}\int d^2z\, & Y^A Y^B E_{AB}^{(1,1)} + Y^A\nabla Y^B C_{AB}^{(0,1)} + Y^A \overline{\nabla} Y^B D_{AB}^{(1,0)} + \widehat{d}_\alpha Y^A G_A^{\alpha(0,1)} +\\ & + \overline{J}_1^I Y^A I_{IA}^{(1,0)} + \overline{J}_2^I K_I^{(1,0)} + J_1 Y^A L_A^{(0,1)} + J_2 M^{(0,1)} + N_1^{ab} Y^A O_{Aab}^{(0,1)} +\\ & + N_2^{ab} P_{ab}^{(0,1)} + J_1 \overline{J}_1^I Q_I^{(0,0)} + N_1^{ab}\overline{J}_1^I R_{Iab}^{(0,0)} + \widehat{d}_\alpha \overline{J}_1^I S_I^{\alpha(0,0)} + \mathcal{O}(\mathcal{Q}^3)\,, \end{aligned} \tag{4.3}$$

where the superfields $C,\ldots,S$ depend on the background superfields and they can be obtained by using the expansions defined in the previous section, but we will not need all of them. The superscripts in these superfields indicate their conformal weights.

As we said before, the effective action is determined by integrating out the variables $\mathcal{Q}$, that is

$$e^{-S_{\text{eff}}} = e^{-S_0}\int D\mathcal{Q}\, e^{-\widehat{S}}\left[1 - \widehat{I} + \frac{1}{2}\widehat{I}^2 + \cdots\right].$$

Remember that the lost of conformal invariance comes from the UV divergences of the Feynman diagrams [9]. There are two types of diagrams which lead to UV divergences in this path integration. A tadpole diagram, which is formed in the single contractions in $\widehat{I}$ of this expansion, and a 'fish' diagram, which is formed by double contractions in the $\widehat{I}^2$ term.

It should be noticed that the effective action should be given by a conformal weight $(1,1)$ density. Therefore, the only terms will contribute to it are those formed by single contraction of Y's in the term with $E_{AB}^{(1,1)}$ in the action and from double contractions in Y's between $Y^A\nabla Y^B C_{AB}^{(0,1)}$ with $Y^A\overline{\nabla}Y^B D_{AB}^{(1,0)}$ and $Y^A\overline{\nabla}Y^B D_{AB}^{(1,0)}$ with $\widehat{d}_\alpha Y^A G_A^{\alpha(0,1)}$ terms in the action. Note that the OPE's between the gauge and ghost currents (3.4), (3.5) provide the right conformal weights. Then, the double contractions between $\overline{J}_1^I Y^A I_{IA}^{(1,0)}$ with $\widehat{d}_\alpha \overline{J}_1^I S_I^{\alpha(0,0)}$ and between $N_1^{ab}\overline{J}_1^I R_{Iab}^{(0,0)}$ with itself will also contribute.

4.1 Computation of the one-loop UV divergence

The single contraction between Y's in the term with $E^{(1,1)}_{AB}$ in the action leads to the divergence

$$-\frac{1}{2\pi}\int d^2z\ \eta^{ab}E^{(1,1)}_{ab}\log\Lambda\,,$$

where Λ is the momentum cut-off.[5] The double contraction between $Y^A\nabla Y^B C^{(0,1)}_{AB}$ with $Y^A\overline{\nabla}Y^B D^{(1,0)}_{AB}$ leads to

$$\frac{1}{2\pi}\int d^2z\ C^{(0,1)}_{ab}D^{(1,0)}_{cd}\eta^{a[c}\eta^{d]b}\log\Lambda\,.$$

The double contraction between $Y^A\overline{\nabla}Y^B D^{(1,0)}_{AB}$ with $\widehat{d}_\alpha Y^A G^{\alpha(0,1)}_A$ is

$$\frac{1}{2\pi}\int d^2z\ \eta^{ab}(D^{(1,0)}_{\alpha a}-D^{(1,0)}_{a\alpha})G^{\alpha(0,1)}_b\log\Lambda\,.$$

The double contraction between $\overline{J}^I_1 Y^A I^{(1,0)}_{IA}$ with $\widehat{d}_\alpha\overline{J}^I_1 S^{\alpha(0,0)}_I$ gives

$$-\frac{1}{2\pi}\int d^2z\ \overline{J}^I f^{JK}{}_I I^{(1,0)}_{J\alpha}S^{\alpha(0,0)}_K\log\Lambda\,.$$

The double contraction between $N^{ab}_1\overline{J}^I_1 R^{(0,0)}_{Iab}$ with itself is

$$-\frac{1}{2\pi}\int d^2z\ \overline{J}^I N^{ab}f^{JK}{}_I R^{(0,0)}_{Jc[a}R^{(0,0)c}_{Kb]}\log\Lambda\,.$$

In summary, the one loop divergence coming from integrating out the quantum fluctuations is

$$\begin{aligned}S_\Lambda=\frac{1}{2\pi}\int d^2z\Big[&-\eta^{ab}E^{(1,1)}_{ab}+\eta^{a[c}\eta^{d]b}C^{(0,1)}_{ab}D^{(1,0)}_{cd}+\eta^{ab}D^{(1,0)}_{[\alpha a]}G^{\alpha(0,1)}_b-\\ &-\overline{J}^I f^{JK}{}_I I^{(1,0)}_{J\alpha}S^{\alpha(0,0)}_K-\overline{J}^I N^{ab}f^{JK}{}_I R^{(0,0)}_{Jc[a}R^{(0,0)c}_{Kb]}\Big]\log\Lambda\,.\end{aligned}\tag{4.4}$$

4.2 The one-loop beta functions

The one-loop effective will be S_Λ plus the Fradkin-Tseytlin contribution. As we mention in the introduction, we are studying the one-loop sigma model at tree-level in the world-sheet. That is we evaluate (2.2) on the sphere, then the world-sheet metric takes the form $\Lambda dz d\bar{z}$. Finally the effective action at the one-loop level will be

$$S_{\text{eff}}=S_0+S_\Lambda+\frac{1}{2\pi}\int d^2z\left[\nabla\overline{\Pi}^A\nabla_A\Phi+\overline{\Pi}^A\Pi^B\nabla_B\nabla_A\Phi\right]\log\Lambda\,.\tag{4.5}$$

The beta functions are defined as the Λ dependent factor of every independent coupling of the effective action and, in order to deal with a theory that is scale independent (such a conformal theory), we need that this dependence to be zero. All these couplings are conformal weights $(1,1)$ constructed out of the products formed in

$$(\Pi^A,d_\alpha,J,N^{ab})\times(\overline{\Pi}^c,\overline{J}^I)\,,$$

[5]One can show that $\log|0|^2=-\log\Lambda$. It is also useful to know that $\int d^2z/|z|^2=2\pi\log\Lambda$.

since $\overline{\Pi}^\alpha$ is not independent. In fact by varying the action (2.1) respect to d_α we obtain $\overline{\Pi}^\alpha = -\overline{J}^I W_I^\alpha$. Also we need the string equations equations of motion to express $\nabla\overline{\Pi}^A$ in terms of the couplings. They come form (2.1) by varying respect to ρ, λ, ω and Z^M as we will show in the next section.

5. The vanishing of the beta functions

Now we can write the equations coming from the vanishing of the beta functions associated to each independent coupling of the sigma model action. Before this, it is convenient to use the set of constraints on the the superfields given in [6]. Namely, we use $H_{\alpha\beta\gamma} = T_{\alpha\beta}{}^\gamma = 0$ and $T_{\alpha\beta}{}^a = \gamma^a_{\alpha\beta}$ besides all the constraints (2.4) and (2.5). These constraints are fixed using the scale and one of the local Lorentz invariances. Note that the remaining local symmetries are those of D=10 N=1 supergravity. Since the fermionic scale invariance is fixed, the R_{AB} superfield is no longer an invariant curvature, and its connections will appear explicitly in the following calculations. The first thing we note is that the constraint $T_{a\alpha}{}^\alpha = 0$ implies that $\Omega_a = 0$. Also, as it was showed in [6], the Bianchi identity $(\nabla T)_{\alpha\beta\gamma}{}^a = 0$[6] determines $T_{\alpha ab} = 2(\gamma_{ab})_\alpha{}^\beta\Omega_\beta$. Since it will be used later, it is useful to write R_{AB} in terms of other fields

$$R_{ab} = T_{ab}{}^\alpha\Omega_\alpha\,, \qquad R_{a\beta} = -R_{\beta a} = \nabla_a\Omega_\beta\,, \qquad R_{\alpha\beta} = \nabla_{(\alpha}\Omega_{\beta)}\,.$$

Also, it is not difficult to show that the Bianchi identity $(\nabla H)_{ab\alpha\beta} = 0$ implies $T_{abc} + H_{abc} = 0$.

Now we write the background superfields needed to determine the beta functions according to (4.5). We use the simplifications derived in the previous paragraph for the background superfields. After the superspace expansion described in the section 3, it is not difficult to get

$$\begin{aligned}
-\eta^{ab}E^{(1,1)}_{ab} = & -\frac{1}{2}d_{0\alpha}\overline{J}^I_0\nabla^2 W_I^\alpha - \frac{1}{2}J_0\overline{J}^I_0\nabla^2 U_I - \frac{1}{4}N_0^{ab}\overline{J}^I_0\nabla^2 U_{Iab} - \\
& - \frac{1}{2}d_{0\alpha}\overline{\Pi}^A(\nabla^a T_{aA}{}^\alpha + T_{Aa}{}^B T_{Bb}{}^\alpha\eta^{ab}) - \frac{1}{2}\Pi^A\overline{J}^I_0(\nabla^a F_{IaA} + T_{Aa}{}^B F_{IB}{}^a) - \\
& - \frac{1}{2}J_0\overline{\Pi}^A(\nabla^a R_{aA} + T_{Aa}{}^B R_B{}^a) - \frac{1}{4}N_0^{ab}\overline{\Pi}^A(\nabla^c R_{cAab} + T_{Ac}{}^B R_B{}^c{}_{ab}) - \\
& - \frac{1}{4}\Pi^A\overline{\Pi}^B\Big(\nabla^a H_{aBA} + T_{Aa}{}^C H_{CB}{}^a(-1)^{AB} - \\
& \qquad - T_{Ba}{}^C H_{CA}{}^a + 2T_{Aa}{}^c T_{Bb}{}^d\eta_{cd}\eta^{ab}(-1)^{AB}\Big) + \\
& + \frac{1}{4}\Pi^{(A}\overline{\Pi}^{a)}\eta_{ab}(\nabla^c T_{Ac}{}^b - T_{Ac}{}^C T_{Cd}{}^b\eta^{cd} + R_{Acd}{}^b\eta^{cd})\,,
\end{aligned} \tag{5.1}$$

where we have to use $\overline{\Pi}^\alpha = -\overline{J}^I W_I^\alpha$. The remaining terms in (4.5) are obtained from

$$\begin{aligned}
C^{(0,1)}_{ab} &= \frac{1}{2}\overline{J}^I(F_{ab} + W_I^\alpha T_{\alpha ab})\,, \\
D^{(1,0)}_{cd} &= \frac{1}{2}d_\alpha T_{cd}{}^\alpha + \frac{1}{2}JR_{cd} + \frac{1}{4}N^{ef}R_{cdef} - \frac{1}{2}\Pi^A T_{Acd}\,,
\end{aligned}$$

[6] Recall our notation (2.10).

$$
\begin{aligned}
D^{(1,0)}_{[\alpha a]} &= JR_{\alpha a} + \frac{1}{2}N^{ef}R_{\alpha aef}\,,\\
G^{\alpha(0,1)}_b &= \overline{J}^I\nabla_b W^\alpha_I - \overline{\Pi}^d T_{db}{}^\alpha\,,\\
I^{(1,0)}_{J\alpha} &= -d_\beta\nabla_\alpha W^\beta_J - \Pi^A F_{JA\alpha} + J\nabla_\alpha U_J + \frac{1}{2}N^{ef}\nabla_\alpha U_{Jef}\,,\\
S^{\alpha(0.0)}_K &= W^\alpha_K, \quad R^{(0,0)}_{Jab} = \frac{1}{2}U_{Jab}\,.
\end{aligned}
$$

As we said before, we need the superstring equations of motion to determine the contribution $\nabla\overline{\Pi}^A$ in (4.5). After the variation of the (2.1) respect to all the world-sheet field, it is not difficult to obtain

$$
\begin{aligned}
\nabla\overline{\Pi}_a =\;& \Pi^b\overline{\Pi}^c T_{abc} + \Pi^\alpha\overline{\Pi}^b T_{a\alpha b} - \Pi^A\overline{J}^I F_{IAa} - d_\alpha\overline{\Pi}^b T_{ba}{}^\alpha - d_\alpha\overline{J}^I\nabla_a W^\alpha_I -\\
&- J\overline{\Pi}^b R_{ba} - \frac{1}{2}\overline{\Pi}^b R_{bacd} + J\overline{J}^I(\nabla_a U_I + W^\gamma_I R_{\gamma a}) + \frac{1}{2}N^{bc}\overline{J}^I(\nabla_a U_{Ibc} + W^\gamma_I R_{\gamma abc})\,,
\end{aligned}
$$

and

$$
\nabla\overline{\Pi}^\alpha = -\overline{J}^I\Pi^A\nabla_A W^\alpha_I - d_\beta\overline{J}^I f^{JK}{}_I W^\alpha_J W^\beta_K + J\overline{J}^I f^{JK}{}_I W^\alpha_J U_K + \frac{1}{2}N^{ab}\overline{J}^I f^{JK}{}_I W^\alpha_J U_{Kab}\,.
$$

With all this information, we can calculate the terms contributing to (4.5). Now we write the beta function associated to every independent coupling of the effective action. They are separated in two groups. The first group of equations is the SUGRA sector

$$
\begin{aligned}
\nabla_a\nabla_b\Phi - \frac{1}{2}\mathcal{R}_{ab} + \frac{1}{2}T_{ca}{}^\alpha T_{b\alpha}{}^c &= 0\,,\\
\nabla^c T_{abc} - 2T_{abc}\nabla^c\Phi + 2T_{ab}{}^\alpha\nabla_\alpha\Phi &= 0\,,\\
\nabla^b T_{\alpha ba} + R_{\alpha cda}\eta^{cd} + T_{ab}{}^\beta\gamma^b_{\beta\alpha} + 4\nabla_a\nabla_\alpha\Phi &= 0\,,\\
\nabla^b T_{ba}{}^\alpha - T_{bc}{}^\alpha T_a{}^{bc} + 2T_{ab}{}^\alpha\nabla^b\Phi &= 0\,,\\
\nabla^b R_{ba} - T_{abc}R^{bc} - T_{ab}{}^\alpha R_\alpha{}^b + 2R_{ab}\nabla^b\Phi &= 0\,,\\
\nabla^b R_{bafg} - T_{abc}R^{bc}{}_{fg} - T_{ab}{}^\alpha R_\alpha{}^b{}_{fg} + 2R_{abfg}\nabla^b\Phi &= 0\,,
\end{aligned}
\tag{5.2}
$$

where $\mathcal{R}_{ab} = R^c{}_{acb}$ is the Ricci tensor which is not symmetric. The second group of equations is the SYM sector

$$
\begin{aligned}
\gamma^a_{\alpha\beta}\nabla_a W^\beta_I = 2(\gamma^a_{\alpha\beta}\nabla_a\Phi)W^\beta_I + 2\nabla_\alpha(\nabla_\beta\Phi W^\beta_I)\,,&\\
\nabla^b F_{Iba} - 2f^{JK}{}_I F_{Ja\alpha}W^\alpha_K + 2F_{Iab}\nabla^b\Phi - 2\nabla_a(\nabla_\alpha\Phi W^\alpha_I) +&\\
+\frac{1}{2}W^\alpha_I(\nabla^b T_{\alpha ba} + T_{ab}{}^\beta\gamma^b_{\beta\alpha} + R_{\alpha cda}\eta^{cd}) &= 0\,,\\
\nabla^2 W^\alpha_I - F^{ab}_I T_{ab}{}^\alpha + 2(\nabla_a W^\alpha_I)\nabla^a\Phi - 2f^{JK}{}_I(\nabla_\beta W^\alpha_J + W^\alpha_J\nabla_\beta\Phi)W^\beta_K &= 0\,,\\
\nabla^2 U_I - F^{ab}_I R_{ab} + W^\beta_I(2R_{a\beta}\nabla^a\Phi - \nabla^a R_{a\beta}) + 2R_{a\alpha}\nabla^a W^\alpha_I -&\\
- 2(\nabla_a U_I)\nabla^a\Phi + 2f^{JK}{}_I(\nabla_\alpha U_J - U_J\nabla_\alpha\Phi)W^\alpha_K &= 0\,,\\
\nabla^2 U_{Ifg} - F^{ab}_I R_{abfg} + W^\beta_I(2R_{a\beta fg}\nabla^a\Phi - \nabla^a R_{a\beta fg}) + 2R_{a\alpha fg}\nabla^a W^\alpha_I -&\\
-2(\nabla_a U_{Ifg})\nabla^a\Phi + 2f^{JK}{}_I(\nabla_\alpha U_{Jfg} - U_{Jfg}\nabla_\alpha\Phi)W^\alpha_K + f^{JK}{}_I U_{Ja[f}U_{Kg]}{}^a &= 0\,.
\end{aligned}
\tag{5.3}
$$

We need to verify that the constraints given in [6] plus the use of the Bianchi identities (2.10) allow to verify the equations (5.2) and (5.3). It is a tedious but direct job, we follow the idea of the calculation done in the reference [10].

5.1 The SUGRA sector

Now it will be shown that the SUGRA set of equations (5.2) are implied by the use of the constraints (2.4), (2.5) and the use of the Bianchi identities. First we note that it is satisfied

$$\gamma^b_{\alpha\beta} T_{ba}{}^{\beta} = 8R_{a\alpha}\,. \tag{5.4}$$

To show this, we see that $R_{\alpha abc}$ can be written, by using the Bianchi identity $(\nabla T)_{\alpha ab}{}^{c} = 0$, as

$$R_{\alpha abc} = T_{a[b}{}^{\beta}(\gamma_{c]})_{\beta\alpha} - 2(\gamma_{bc})_{\alpha}{}^{\beta} R_{a\beta}\,,$$

we plug this into the Bianchi identity $(\nabla T)_{a\alpha\beta}{}^{\beta} = 0$ to verify (5.4). Now it is trivial to show that the third equation in (5.2) is satisfied.

Now we will show that the second equation in (5.2) is also satisfied. Consider the Bianchi identity $(\nabla T)_{abc}{}^{c} = 0$, it implies

$$\nabla^c T_{abc} + 4\Omega_\alpha T_{ab}{}^{\alpha} - 16\Omega_\alpha \gamma^{\alpha\beta}_{[a} R_{b]\beta} + \eta^{cd}(R_{acbd} - R_{bdac}) = 0\,.$$

From the Bianchi identity $(\nabla T)_{\alpha ab}{}^{\beta} = 0$ one can obtain

$$R_{abcd} = -\frac{1}{8}\nabla_\alpha((\gamma_{cd})_\beta{}^{\alpha} T_{ab}{}^{\beta}) + \frac{1}{8}(\gamma_{cd})_\beta{}^{\alpha} T_{\alpha[a}{}^{e} T_{b]e}{}^{\beta}\,, \tag{5.5}$$

which allows to write $\eta^{cd}(R_{acbd} - R_{bdac})$ and, after plugging it into the above equation one obtains

$$\nabla^c T_{abc} + \gamma^{\alpha\beta}_a \nabla_\alpha R_{b\beta} - \gamma^{\alpha\beta}_b \nabla_\alpha R_{a\beta} + 2\Omega_\alpha(\gamma_a\gamma_{bc} - \gamma_b\gamma_{ac})^{\beta\alpha} R^c{}_{\beta} = 0\,,$$

finally if one uses $R_{a\alpha} = \nabla_a\Omega_\alpha$ and commutes the derivatives in $\nabla_\alpha R_{b\beta}$ and in $\nabla_\alpha R_{a\beta}$ one can arrive to the second equation in (5.2). We can verify the first equation in (5.2) by constructing the Ricci tensor $\mathcal{R}_{ab} = R^c{}_{acb}$ from (5.5).

It remains to verify the last three equations in (5.2). Note that the fourth and the fifth equations are equivalent by recalling the relation $R_{ab} = T_{ab}{}^{\alpha}\Omega_\alpha$. Without using the previous equations, the derivation of the fourth equation is more involved. Using the Bianchi indentities $(\nabla R)_{[\alpha ab]\beta}{}^{\gamma} = 0$, $(\nabla T)_{c\alpha\beta}{}^{\gamma} = 0$ and $(\gamma_a)^{\alpha\beta} R_{\alpha\beta\gamma}{}^{\delta} = -2(\gamma_a)^{\alpha\beta} R_{\gamma\alpha\beta}{}^{\delta}$, that follows from $(\nabla T)_{\alpha\beta\gamma}{}^{\delta} = 0$ we show that

$$\begin{aligned}(\gamma^a)^{\alpha\beta}(\nabla_\alpha R_{ab\beta}{}^{\gamma} - T_{\alpha[a}{}^{e} R_{b]e\beta}{}^{\gamma}) - 8T_b{}^{cd} T_{cd}{}^{\gamma} + 2T_{eb}{}^{\gamma}\nabla^e\Phi + 8\nabla^e T_{eb}{}^{\gamma} - \\ - \frac{1}{8}(\gamma^a)^{\alpha\beta}(\gamma^{cd})_\rho{}^{\gamma} R_{\alpha\beta cd} T_{ab}{}^{\rho} = 0\,,\end{aligned}$$

and after working out the first and the last terms by using (5.5) and the Bianchi identity $(\nabla T)_{\alpha\beta a}{}^{b} = 0$, we see that they give exactly the remaining terms, proving the fourth equation. Finally the last equation is satisfied by using (5.5) and the remaining equations in (5.2).

We can see that the first equation is the graviton equation of motion, the second is the equation for the antisymmetric tensor. Contracting the third equation with $(\gamma^a)^{\alpha\beta}$ we get the dilatino equation of motion and contracting with $(\gamma^{ac})_\beta{}^{\alpha}$ we get the gravitino equation.[7] The last three equations are redundant, they can be obtained from the others. These results prove the claim in [6] that the classical BRST invariance is equivalent to quantum conformal invariance at 1-loop level.

5.2 The SYM sector

The verification of the SYM equations is the following. Let us first consider the first equation in (5.3). To verify that it is implied by the Bianchi identities it is necessary to relate the field strength F with the superfield U. The Bianchi identity $(\nabla F)_{a\alpha\beta} = 0$ implies

$$U_I + \Omega_\alpha W_I^\alpha = 0\,,$$
$$F_{Iab} = U_{Iab} + 2(\gamma_{ab})_\alpha{}^\beta \Omega_\beta W_I^\alpha\,.$$

Besides, it will be necessary to know the spinorial derivative on this superfield. The Bianchi identity $(\nabla F)_{\alpha ab} = 0$ determines

$$\nabla_\alpha F_{Iab} = -\nabla_{[a} F_{Ib]\alpha} + T_{\alpha[a}{}^c F_{Ib]c} - T_{ab}{}^c F_{Ic\alpha}\,.$$

The gluino equation can be obtained from the identity

$$\{\nabla_\alpha, \nabla_\beta\} W_I^\beta = -T_{\alpha\beta}{}^a \nabla_a W_I^\beta + R_{\alpha\gamma\beta}{}^\gamma W_I^\beta\,,$$

and using the constraint equation for $\nabla_\alpha W_I^\beta$ in (2.5). It is also necessary to determine $R_{\alpha\gamma ab}(\gamma^{ab})_\beta{}^\gamma$ from the Bianchi identity $(\nabla T)_{\alpha\beta a}{}^b = 0$:

$$R_{\alpha\gamma ab}(\gamma^{ab})_\beta{}^\gamma = -180\nabla_\alpha\Omega_\beta + 2(\gamma^{ab})_\alpha{}^\gamma(\gamma_{ab})_\beta{}^\delta \nabla_\delta\Omega_\gamma - \gamma^{abc}_{\alpha\beta} T_{abc} - 384\Omega_\alpha\Omega_\beta\,,$$

where we have used the identity $(\gamma^{ab})_\alpha{}^\rho(\gamma_{ab})_\beta{}^\sigma \Omega_\rho\Omega_\sigma = 6\Omega_\alpha\Omega_\beta$. Doing all this we obtain

$$-\frac{7}{2}\gamma^a_{\alpha\beta}\nabla_a W_I^\beta - 28\nabla_\alpha U_I - \left[R_{\alpha\beta} + \frac{1}{2}(\gamma^{ab})_\alpha{}^\rho(\gamma_{ab})_\beta{}^\sigma R_{\rho\sigma}\right] W_I^\beta = 0\,,$$

where $R_{\alpha\beta} = \nabla_{(\alpha}\Omega_{\beta)}$. The Bianchi identity $R_{(\alpha\beta\gamma)}{}^\gamma = 0$ allows us to finally write

$$\gamma^a_{\alpha\beta}\nabla_a W_I^\beta = -8(\nabla_\alpha U_I + R_{\alpha\beta} W_I^\beta)\,.$$

which takes the form of the first equation in (5.3) if we recall (2.11).

Now we satisfy the second equation in (5.3). The idea is to start with the knowledge of the commutator $[\nabla_b, \nabla_\alpha] W_I^\beta$, which can be obtained from (2.7), and multiply by $(\gamma^{ab})_\beta{}^\alpha$, and use the first equation in (5.3) together with the third equation in (5.2). After a tedious, but direct calculation, the second equation can be showed to be satisfied. Similarly for the remaining equations in the SYM sector can be verified as consequence of the first two equations in (5.3). The third equation is obtained by applying $(\gamma^b)^{\alpha\gamma}\nabla_b$ in the first equation in (5.3), while the last two equations can be obtained by acting with ∇_β on the third equation and after commuting it with ∇^2. Then, the fourth equation is obtained by contracting with δ_α^β and we get the fifth equation and by multiplying with $(\gamma_{ef})_\alpha{}^\beta$.

[7] Remember that at linearized level $T_{ab}{}^\alpha \approx \partial_{[a}\psi_{b]}{}^\alpha$, where $\psi_a{}^\alpha$ is the gravitino.

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